# CONE ARRAY OF FOURIER LENSES FOR CONTOURING APPLICATIONS 

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The paper investigates an application of Fourier transform lenses to surface topography detection. The field is illuminated in parralel using an array of three different wavelength laser diodes and relative phase alteration is read out between cross-correlated electronic pixels.Diffraction theory of image formation is used to interpret the modula tion in contrast. Speckle sensitivity to wavelength provides the basic technique for noise filtering.

## 1 INITRODUCTION

A variety of procedures for optical 3D sensing are available today.Among them,diffracti ve techniques offer well known benefits over similar approaches like triangulation,grid or moiré projection or modulation interferometry. There are no image missing points due to separate vewing directions of the object, speed limitations in data aquisition (one point or line framed at a time) or readout degeneracy related to the inherent periodici ty of the fringe pattern.
However it is also known that, even diffractive methods imply tradeoff restrictions such as contouring accuracy vs. speed and depth of focus vs.lateral resolution. Among other factors, image resolution is determined by pixel size and detector nonlinearity. In con ventional iqaging systems high lateral resolution $1 / \delta x$ limits the depth measurement ran ge $\Delta z$ since ${ }^{\prime}$ :

$$
\begin{equation*}
\Delta z=4 \delta x^{2} / \lambda \tag{1}
\end{equation*}
$$

The arrangement presented below is meant to improve performance capabilities of diffrac tive 3 D sensing techniques. It has been shown that proximity sensors having multiple light sources yield good results in terms of accuracy and measurement speed . Based on this "geometrical redundancy",our approach employs a combination of high resolution Fou rier transform lenses working in conjunction with an equal number of coherent sources. To eliminate crosstalk, the ring array of sources consists of three distinct wavelength laser diodes located $120^{\circ}$ apart. Lenses are organized in a typical"canonical optical pro cessor"configuration (see fig. 1).Telecentric beams projected onto the object are converted into electronic signals and compared via differential readings.To obtain subpixel resolution a diffraction limited spot is required which,in turn,demands a large N.A. , small field size and small depth of focus.
Since optics is stationary, a motorized X-Y stage may be used to cover the full object extension. One of the three illumination paths incorporates a bandpass filter wheel. As explained below, frequency modulation is provided to asses surface roughness in a simi lar way with speckle metrology. Drawbacks of the arrangement include the need for three monochromatic sources, well corrected optics and fairly complex electronics for amplification and signal processing. The setup is suitable for applications involving opaque and highly reflective objects having various degrees of surface defects and irregularity. Topography detection becomes erroneous for absorptive or extremely diffuse surfaces.

## 2 SCHEMATICS DESCRIPTION

Fig. 1 shows a vertical representation for one of the three paths.IP specifies the input plane containing a circular aperture or a rectangular grid ( $x, y$ ) of spatial frequencies ( $f{ }_{x^{\prime}} f{ }_{y}$ ), FP stands for the frequency plane where appropiate stops or masks are located, OP ${ }^{\prime}$ is ${ }^{Y}$ the output plane. Because the arrangement is symmetrical to the frequency plane, both coma and distorsion are cancelled as long as FT lenses are identical.As in any other afocal system, EP is the entrance pupil while EP' stands for the exit pupil. In the picture the frequency plane has a central photodetector dot serving also as a high pass filter for contrast enhancement. The frequency plane contains the far-field diffraction pattern of the input aperture. Surface topography is detected in the output plane. There is a focussing detector placed at the best focus location corresponding to a selected point-object on axis. Filter $F_{0}$ is recommended to prevent any interference be tween paths.Position of the focussing detector can be slightly shifted using a micrometric drive or a differential screw (MD).FW is the bandpass filter wheel, $\mathrm{C}_{1,2}$ are two comparators and $\mu \mathrm{P}$ represents the system's microprocessor.

## 3 THEORY OF OPERATION

If the object is a perfectly flat and fully reflective surface, the complex amplitude distribution recorded at OP is given by the inverse Fourier transform of the far-field diffraction pattern associated with the input aperture:

$$
\begin{equation*}
E(x, y)=F T^{-1}[E(s, t)] \tag{2}
\end{equation*}
$$

where s,t define coordinates in the frequency plane.Local departures from the flat surface generate OPD changes in the reflected beam which alter the diffraction integral. Surface modulation can be translated as appropiate deformations of the wavefront exiting the system through EP'.Hence the theory of operation amounts to the rela̧tionship between the diffraction integral and the aberrated wave returned by the object. ${ }^{\text {b }}$
Assuming that no scatter is present, a differential OPD increment can be derived from fig.2.Ray (2) hits the elementary surface step earlier than ray (1) due to segment $\delta 1_{1}$. The segment EF equals the resulting path shift between ( $1^{\prime}$ ) and (2') which reads, after simple calculations:

$$
\begin{equation*}
\phi(x)=2 \sin \alpha \int\left(1+\tilde{z}_{x}^{\prime, 2}\right)^{1 / 2} \cos \left(\operatorname{arctg} \tilde{z}_{x}^{\prime}\right) d x \tag{3}
\end{equation*}
$$

where $\tilde{z}_{x}^{\prime}=\partial \tilde{z} / \partial \tilde{x}$ is the local surface gradient. The output normalized intensity is given by the Strehl factor:

$$
\begin{gather*}
i\left(x^{\prime}, y^{\prime}\right)=I\left(x^{\prime}, y^{\prime}\right) / I^{*}=\bar{\pi}^{2} \int_{0}^{1} \int_{0}^{2 \pi} \exp i\left[k \Phi(\rho, \theta)-v \rho \cos (\theta-\psi)-1 / 2 u \rho^{2}\right] \rho d \rho d \theta  \tag{4}\\
, \quad,
\end{gather*}
$$

in which $X, Y$ are coordinates in the output plane and $\rho, \theta$ represent polar coordinates in the exit pupil plane. In the same expression $k$ stands for the wavevector and $u, v$ are the so-called optical coordinates:

$$
\begin{gather*}
u\left(x^{\prime}, y^{\prime}\right)=k(a / R)^{2} z^{\prime}  \tag{5}\\
v\left(x^{\prime}, y^{\prime}\right)=k(a / R)\left(x^{\prime 2}+y^{\prime 2}\right)^{1 / 2}
\end{gather*}
$$

where a is pupil radius and $\left.R_{=\left(x^{\prime}\right.}^{2}+y^{\prime 2}+z^{\prime 2}\right) . \Phi(\rho, \theta)$ can be, related to (3) using transform relationships between coordinates in conjugates planes.

For a fairly smooth surface (small aberrations), the normalized intensity at the centre of reference sphere measured in the neighbourghood of the focus spot yields:

$$
\begin{equation*}
i\left(x^{\prime}, Y^{\prime}\right) \sim 1-k^{2}(\Delta \Phi)^{2} \tag{6}
\end{equation*}
$$

where:

$$
\begin{equation*}
(\Delta \Phi)^{2}=\bar{\Phi}^{2}-(\bar{\Phi})^{2} \quad \text { (mean square wavefront deform.) } \tag{7}
\end{equation*}
$$

Note also from (3) that the differential phase shift depends on sin $\alpha$ which means that better intensity resolution requires larger incidence angles. This remains true only when overall scattering effects due to surface texture are negligible. The basic setup can be slightly modified to deliver color encoded depth information. Because good lateral resolution calls for small depth of focus (rel. (1)), it makes sense to use each of the three wavelengths for collecting topography data from different depth levels.Hence the surface is "sliced" into three depth increments and each monochromatic focussing detector is set to sample a selective depth range. Out of focus rays are electonically blocked and overall image reconstruction reduces to the boolean superposition of the three independent frames.(fig.3)

## 4 NOISE FILTERING

As in any other 3D optical sensing approach, surface roughness and reflectivity variati ons across are the main noise contributors. Considerable effort has been dedicated to the subject of optimal noise filtering. It has been reported for instance ${ }^{4}$ that large fluctuations in object reflectivity (greater than four decade range of peak intensities) requires modulation in the illumination level to maintain subpixel resolution. It is also recommended to use an analog low-pass filter with a cutoff frequency of $2 \sigma$, where a calibration Gaussian stripe of variance $\sigma^{2}$ is being projected onto the detector. As far as speckle is concerned, our approach takes adyantage of the wavelength sensitivity of the specular pattern reflected by the object ${ }^{5}$. If the surface represents a flat diffuser of $r-m-s$ roughness $h$ illuminated in parralel coherent light, the pattern deco rrelates with wavelength as:

$$
\begin{equation*}
\lambda^{2} / \Delta \lambda \sim 4 \pi h \tag{8}
\end{equation*}
$$

where $\Delta \lambda$ is the wavelength modulation required to produce a perceivable change. When the object has a composite shape represented by a basic profile and a texture:

$$
\begin{equation*}
z(\tilde{x})=z_{0}(\tilde{x})+h(\tilde{x}) \tag{9}
\end{equation*}
$$

the far-field scattered amplitude is given, in general, by:

$$
\begin{equation*}
E(\hat{x}, \hat{z}, k)=e^{-i k r} \int \exp i k\left[\tilde{x}\left(\sin \alpha+\sin \alpha^{\prime}\right)+h(\tilde{x})\left(\cos \alpha+\cos \alpha^{\prime}\right)\right] \tilde{d x} \tag{10}
\end{equation*}
$$

where $\hat{x}, \hat{z}$ define the observation point, $r$ the position yector modulus $r=\left(\hat{x}^{2}+\hat{z}^{2}\right)^{1 / 2}$ and $\alpha$ ' the observation angle measured with respect to the $z$-direction. For all practical purposes (10) can be modified to explicitly highlight shape and roughness contributions:

$$
\begin{equation*}
E(\hat{x}, \hat{z}, \nu) \sim \int h(\tilde{x}) \cdot \exp \left(i 4 \pi \nu z_{0}(\tilde{x}) / c\right) \cdot \exp (i 4 \pi \nu p(\tilde{x}) / c) d \tilde{x} \tag{11}
\end{equation*}
$$

in which $z_{0}(\tilde{x})$ is much more sensitive to wavelength change and $\left.p \tilde{x}\right)$ is the roughness
phase factor.
When performing speckle evaluation,filter $F_{0}$ is flipped out of the way and observation is done in one of the three paths available, preferable the most sensitive to wavelength modulation.

## 5 REFERENCES

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Fig. 2


Fig. 3

